

① 導関数の定義に従って、次の関数の導関数を求めよ。

(1)  $f(x) = (3x+1)^2$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{\{3(x+h)+1\}^2 - (3x+1)^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{6h(3x+1) + 9h^2}{h} \\ &= \lim_{h \rightarrow 0} \{6(3x+1) + 9h\} \\ &= \underline{6(3x+1)} \end{aligned}$$

(2)  $f(x) = \frac{1}{2x-1}$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{1}{h} \left\{ \frac{1}{2(x+h)-1} - \frac{1}{2x-1} \right\} \\ &= \lim_{h \rightarrow 0} \frac{1}{h} \cdot \frac{-2h}{(2x+2h-1)(2x-1)} \\ &= \underline{-\frac{2}{(2x-1)^2}} \end{aligned}$$

(3)  $f(x) = \sqrt{x^2-2}$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{\sqrt{(x+h)^2-2} - \sqrt{x^2-2}}{h} \\ &= \lim_{h \rightarrow 0} \frac{2hx + h^2}{h(\sqrt{(x+h)^2-2} + \sqrt{x^2-2})} \\ &= \underline{\frac{x}{\sqrt{x^2-2}}} \end{aligned}$$

(4)  $f(x) = \frac{x}{x+1}$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{1}{h} \left\{ \frac{x+h}{(x+h)+1} - \frac{x}{x+1} \right\} \\ &= \lim_{h \rightarrow 0} \frac{1}{h} \cdot \frac{h}{(x+h+1)(x+1)} \\ &= \underline{\frac{1}{(x+1)^2}} \end{aligned}$$

② 次の関数を微分せよ。

(1)  $y = -5x^5$

$$y' = \underline{-25x^4}$$

(2)  $y = \frac{1}{7}x^7 - x^5 - 4x^4 - 2x^2 + 2$

$$y' = \underline{x^6 - 5x^4 - 16x^3 - 4x}$$

(3)  $y = x^{-4}$

$$y' = \underline{-4x^{-5}}$$

(4)  $y = -3x^{-7}$

$$y' = \underline{21x^{-8}}$$

(5)  $y = \frac{1}{x^6} = x^{-6}$

$$y' = \underline{-6x^{-7}} = \underline{-\frac{6}{x^7}}$$

(6)  $y = -\frac{1}{6x^8} = -\frac{1}{6}x^{-8}$

$$y' = \underline{\frac{4}{3}x^{-9}} = \underline{\frac{4}{3x^9}}$$

(7)  $y = x^2 + \frac{1}{x^5}$

$$y' = \underline{2x - \frac{5}{x^6}}$$

(8)  $y = \frac{x^5 - x^2 + 1}{x^2}$

$$y' = \underline{3x^2 - \frac{2}{x^3}}$$

(9)  $y = x^{\frac{4}{3}}$

$$y' = \underline{\frac{4}{3}x^{\frac{1}{3}}}$$

(10)  $y = x^{-\frac{1}{4}}$

$$y' = \underline{-\frac{1}{4}x^{-\frac{5}{4}}}$$

(11)  $y = \sqrt[10]{x} = x^{\frac{1}{10}}$

$$y' = \underline{\frac{1}{10}x^{-\frac{9}{10}}} = \underline{\frac{1}{10 \cdot \sqrt[10]{x^9}}}$$

(12)  $y = \sqrt[3]{x^3} = x^{\frac{1}{2}}$

$$y' = \underline{\frac{1}{2}x^{-\frac{1}{2}}} = \underline{\frac{1}{2\sqrt{x}}}$$

(13)  $y = \frac{2}{\sqrt[5]{x}} = 2 \cdot x^{-\frac{1}{5}}$

$$y' = \underline{-\frac{2}{5}x^{-\frac{6}{5}}} = \underline{-\frac{2}{5x^{\frac{6}{5}}}}$$

(14)  $y = \frac{2}{x\sqrt{x}} = 2x^{-\frac{3}{2}}$

$$y' = \underline{-3x^{-\frac{5}{2}}} = \underline{-\frac{3}{x^2\sqrt{x}}}$$

(15)  $y = x^2 \cdot \sqrt[4]{x^3} = x^{\frac{11}{4}}$

$$y' = \underline{\frac{11}{4}x^{\frac{7}{4}}} = \underline{\frac{11}{4x^{\frac{1}{4}}\sqrt{x^3}}}$$

(16)  $y = x \cdot \sqrt[3]{x} + \frac{4}{\sqrt[3]{x^5}}$

$$y' = \underline{\frac{4}{3}x^{\frac{1}{3}} - \frac{5}{2}x^{-\frac{13}{6}}} = \underline{\frac{4}{3\sqrt[3]{x}} - \frac{5}{2x^2\sqrt[6]{x^5}}}$$

③ 次の関数を微分せよ。

(1)  $y=(x-2)(4x+3)$

$$\begin{aligned} y' &= 1 \cdot (4x+3) + (x-2) \cdot 4 \\ &= 8x-5 \end{aligned}$$

(2)  $y=(3x-1)(x+3)$

$$\begin{aligned} y' &= 3(x+3) + (3x-1) \cdot 1 \\ &= 6x+8 \end{aligned}$$

(3)  $y=(x^3-1)(2x^2+3)$

$$\begin{aligned} y' &= 3x^2(2x^2+3) + (x^3-1) \cdot 4x \\ &= 10x^4 + 9x^2 - 4x \end{aligned}$$

(4)  $y=(3x-1)(2x^2-x+1)$

$$\begin{aligned} y' &= 3(2x^2-x+1) + (3x-1)(4x-1) \\ &= 18x^2 - 10x + 4 \end{aligned}$$

(5)  $y=(x^4-2x)(x^3+3)$

$$\begin{aligned} y' &= (4x^3-2)(x^3+3) + (x^4-2x) \cdot 3x^2 \\ &= 7x^6 + 4x^3 - 6 \end{aligned}$$

(6)  $y=(x^2-4x+2)(3x^2-1)$

$$\begin{aligned} y' &= (2x-4)(3x^2-1) + (x^2-4x+2) \cdot 6x \\ &= 12x^3 - 36x^2 + 10x + 4 \end{aligned}$$

(7)  $y=(x-1)(x^3-3x^2+x+3)$

$$\begin{aligned} y' &= 1 \cdot (x^3-3x^2+x+3) + (x-1)(3x^2-6x+1) \\ &= 4x^3 - 12x^2 + 8x + 2 \end{aligned}$$

(8)  $y=(x^2+2x-2)(x^2-x+3)$

$$\begin{aligned} y' &= (2x+2)(x^2-x+3) + (x^2+2x-2)(2x-1) \\ &= 4x^3 + 3x^2 - 2x + 8 \end{aligned}$$

④ 次の関数を微分せよ。

(1)  $y=(3x+1)(x^2-1)$

$$\begin{aligned} y' &= 3(x^2-1) + (3x+1) \cdot 2x \\ &= 9x^2 + 2x - 3 \end{aligned}$$

(2)  $y=(2x^2-3)(x-1)$

$$\begin{aligned} y' &= 4x(x-1) + (2x^2-3) \cdot 1 \\ &= 6x^2 - 4x - 3 \end{aligned}$$

(3)  $y=(2x-1)(x^2-2x-2)$

$$\begin{aligned} y' &= 2(x^2-2x-2) + (2x-1)(2x-2) \\ &= 6x^2 - 10x - 2 \end{aligned}$$

(4)  $y=(2x^2+3x-4)(5x^2-1)$

$$\begin{aligned} y' &= (4x+3)(5x^2-1) + (2x^2+3x-4) \cdot 10x \\ &= 40x^3 + 45x^2 - 44x - 3 \end{aligned}$$

(5)  $y=(x^4-1)(x^2+3x)$

$$\begin{aligned} y' &= 4x^3(x^2+3x) + (x^4-1)(2x+3) \\ &= 6x^5 + 15x^4 - 2x - 3 \end{aligned}$$

(6)  $y=(x^5-x^3+x^2-1)(x+2)$

$$\begin{aligned} y' &= (5x^4-3x^2+2x)(x+2) \\ &\quad + (x^5-x^3+x^2-1) \cdot 1 \\ &= 6x^5 + 10x^4 - 4x^3 + 9x^2 + 4x - 1 \end{aligned}$$

(7)  $y=(2x^2-x-1)(x^2-3x+3)$

$$\begin{aligned} y' &= (4x-1)(x^2-3x+3) + (2x^2-x-1)(2x-3) \\ &= 8x^3 - 21x^2 + 16x \end{aligned}$$

⑤ 次の関数を微分せよ。

(1)  $y = \frac{1}{3x^2-1}$

$$y' = -\frac{6x}{(3x^2-1)^2}$$

(2)  $y = \frac{2}{2x+1}$

$$y' = -\frac{4}{(2x+1)^2}$$

(3)  $y = \frac{2x+3}{x-1}$

$$y' = \frac{2(x-1) - (2x+3) \cdot 1}{(x-1)^2}$$

$$= -\frac{5}{(x-1)^2}$$

(4)  $y = \frac{x-2}{4x+1}$

$$y' = \frac{1 \cdot (4x+1) - (x-2) \cdot 4}{(4x+1)^2}$$

$$= \frac{9}{(4x+1)^2}$$

(5)  $y = \frac{2x}{x^2-2}$

$$y' = \frac{2(x^2-2) - 2x \cdot 2x}{(x^2-2)^2}$$

$$= -\frac{2x^2+4}{(x^2-2)^2}$$

(6)  $y = \frac{x^2-x+2}{x+1}$

$$y' = \frac{(2x-1)(x+1) - (x^2-x+2) \cdot 1}{(x+1)^2}$$

$$= \frac{x^2+2x-3}{(x+1)^2}$$

(7)  $y = \frac{3x+1}{x^3-2x-2}$

$$y' = \frac{3(x^3-2x-2) - (3x+1)(3x^2-2)}{(x^3-2x-2)^2}$$

$$= -\frac{6x^3+3x^2+4}{(x^3-2x-2)^2}$$

⑥ 次の関数を微分せよ。

(1)  $y = \frac{1}{5x-1}$

$$y' = -\frac{5}{(5x-1)^2}$$

(2)  $y = -\frac{5}{4x^2+3}$

$$y' = \frac{40x}{(4x^2+3)^2}$$

(3)  $y = \frac{x-2}{x+1}$

$$y' = \frac{1 \cdot (x+1) - (x-2) \cdot 1}{(x+1)^2}$$

$$= \frac{3}{(x+1)^2}$$

(4)  $y = \frac{2x+1}{x^2-1}$

$$y' = \frac{2(x^2-1) - (2x+1) \cdot 2x}{(x^2-1)^2}$$

$$= \frac{-2(x^2+x+1)}{(x^2-1)^2}$$

(5)  $y = \frac{3x^2-1}{x+1}$

$$y' = \frac{6x(x+1) - (3x^2-1) \cdot 1}{(x+1)^2}$$

$$= \frac{3x^2+6x+1}{(x+1)^2}$$

(6)  $y = \frac{x}{x^3-4x+1}$

$$y' = \frac{1 \cdot (x^3-4x+1) - x(3x^2-4)}{(x^3-4x+1)^2}$$

$$= \frac{-2x^3+1}{(x^3-4x+1)^2}$$

(7)  $y = \frac{x^4-3x+6}{x+1}$

$$y' = \frac{(4x^3-3)(x+1) - (x^4-3x+6) \cdot 1}{(x+1)^2}$$

$$= \frac{3x^4+4x^3-9}{(x+1)^2}$$